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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

2 hours

Examination code

2	2	1	4	–	7	2	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Events A and B are such that $\mathrm{P}(A)=\frac{2}{5}$, $\mathrm{P}(B)=\frac{11}{20}$ and $\mathrm{P}(A|B)=\frac{2}{11}$.

(a) Find $\mathrm{P}(A\cap B)$. *[2]*

(b) Find $\mathrm{P}(A\cup B)$. *[2]*

(c) State with a reason whether or not events A and B are independent. *[2]*

2. *[Maximum mark: 5]*

Solve the equation $8^{x-1} = 6^{3x}$. Express your answer in terms of $\ln 2$ and $\ln 3$.

3. *[Maximum mark: 5]*

(a) Show that the following system of equations has an infinite number of solutions. *[2]*

$$\begin{aligned} x + y + 2z &= -2 \\ 3x - y + 14z &= 6 \\ x + 2y &= -5 \end{aligned}$$

The system of equations represents three planes in space.

(b) Find the parametric equations of the line of intersection of the three planes. *[3]*

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4. *[Maximum mark: 6]*

The roots of a quadratic equation $2x^2 + 4x - 1 = 0$ are α and β.

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$; *[4]*

(b) find a quadratic equation with roots α^2 and β^2. *[2]*

5. *[Maximum mark: 5]*

(a) Sketch the graph of $y = \left|\cos\left(\frac{x}{4}\right)\right|$ for $0 \le x \le 8\pi$. *[2]*

(b) Solve $\left|\cos\left(\frac{x}{4}\right)\right| = \frac{1}{2}$ for $0 \le x \le 8\pi$. *[3]*

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6. *[Maximum mark: 6]*

PQRS is a rhombus. Given that $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{QR} = \boldsymbol{b}$,

(a) express the vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$; *[2]*

(b) hence show that the diagonals in a rhombus intersect at right angles. *[4]*

7. *[Maximum mark: 7]*

Consider the complex numbers $u = 2 + 3\mathrm{i}$ and $v = 3 + 2\mathrm{i}$.

(a) Given that $\frac{1}{u} + \frac{1}{v} = \frac{10}{w}$, express w in the form $a + b\mathrm{i}$, $a, b \in \mathbb{R}$. *[4]*

(b) Find w^* and express it in the form $re^{\mathrm{i}\theta}$. *[3]*

8. *[Maximum mark: 6]*

The function f is defined by

$$f(x) = \begin{cases} 1 - 2x, & x \le 2 \\ \dfrac{3}{4}(x-2)^2 - 3, & x > 2 \end{cases}$$

(a) Determine whether or not f is continuous. *[2]*

The graph of the function g is obtained by applying the following transformations to the graph of f:

a reflection in the y–axis followed by a translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

(b) Find $g(x)$. *[4]*

9. *[Maximum mark: 7]*

The first three terms of a geometric sequence are $\sin x$, $\sin 2x$ and $4\sin x\cos^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Find the common ratio r. *[1]*

(b) Find the set of values of x for which the geometric series $\sin x + \sin 2x + 4\sin x\cos^2 x + \ldots$ converges. *[3]*

Consider $x = \arccos\left(\frac{1}{4}\right)$, $x > 0$.

(c) Show that the sum to infinity of this series is $\frac{\sqrt{15}}{2}$. *[3]*

10. *[Maximum mark: 7]*

Use the substitution $x = a\sec\theta$ to show that $\int_{a\sqrt{2}}^{2a} \frac{\mathrm{d}x}{x^3\sqrt{x^2 - a^2}} = \frac{1}{24a^3}\left(3\sqrt{3} + \pi - 6\right)$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 12]*

(a) Mobile phone batteries are produced by two machines. Machine A produces 60% of the daily output and machine B produces 40%. It is found by testing that on average 2% of batteries produced by machine A are faulty and 1% of batteries produced by machine B are faulty.

(i) Draw a tree diagram clearly showing the respective probabilities.

(ii) A battery is selected at random. Find the probability that it is faulty.

(iii) A battery is selected at random and found to be faulty. Find the probability that it was produced by machine A. *[6]*

(b) In a pack of seven transistors, three are found to be defective. Three transistors are selected from the pack at random without replacement. The discrete random variable X represents the number of defective transistors selected.

(i) Find $\text{P}(X = 2)$.

(ii) **Copy** and complete the following table:

x	0	1	2	3
$\text{P}(X = x)$				

(iii) Determine $\text{E}(X)$. *[6]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 18]*

Given the points A(1, 0, 4), B(2, 3, −1) and C(0, 1, −2),

(a) find the vector equation of the line L_1 passing through the points A and B. *[2]*

The line L_2 has Cartesian equation $\frac{x-1}{3}=\frac{y+2}{1}=\frac{z-1}{-2}$.

(b) Show that L_1 and L_2 are skew lines. *[5]*

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

(c) Find the Cartesian equation of the plane Π_1. *[4]*

The line L_3 has vector equation $\boldsymbol{r}=\begin{pmatrix}3\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}k\\1\\-1\end{pmatrix}$.

The plane Π_2 has Cartesian equation $x+y=12$.

The angle between the line L_3 and the plane Π_2 is 60°.

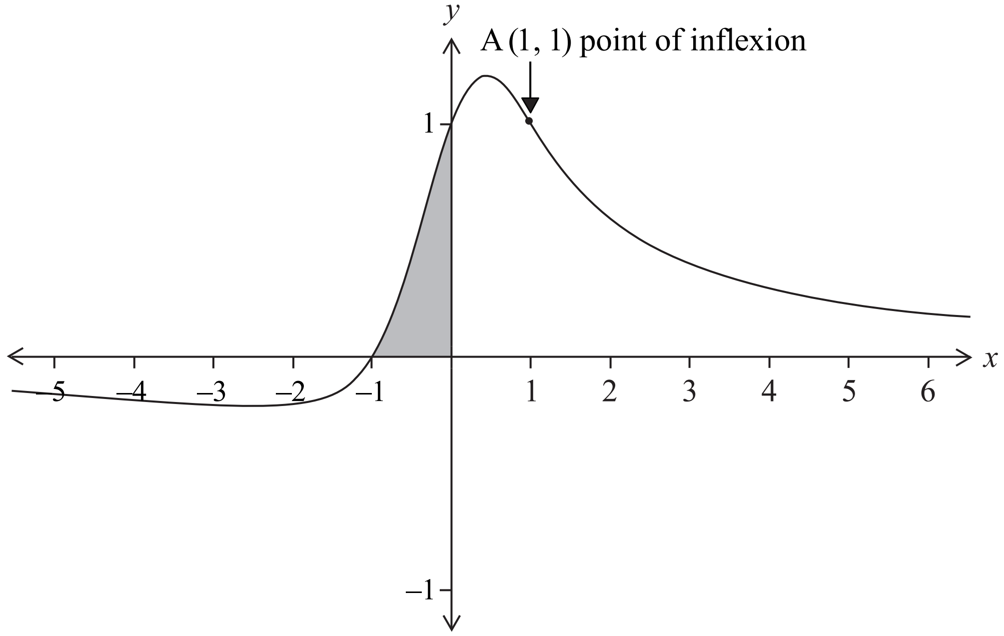
(d) (i) Find the value of k.

(ii) Find the point of intersection P of the line L_3 and the plane Π_2. *[7]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 16]*

The graph of the function $f(x) = \dfrac{x+1}{x^2+1}$ is shown below.



(a) Find $f'(x)$. *[2]*

(b) Hence find the x-coordinates of the points where the gradient of the graph of f is zero. *[1]*

(c) Find $f''(x)$ expressing your answer in the form $\dfrac{p(x)}{(x^2+1)^3}$, where $p(x)$ is a polynomial of degree 3. *[3]*

The point $(1, 1)$ is a point of inflexion. There are two other points of inflexion.

(d) Find the x-coordinates of the other two points of inflexion. *[4]*

(e) Find the area of the shaded region. Express your answer in the form $\dfrac{\pi}{a} - \ln\sqrt{b}$, where a and b are integers. *[6]*

*Do **NOT** write solutions on this page.*

14. *[Maximum mark: 14]*

Consider the following functions:

$$h(x) = \arctan(x), \; x \in \mathbb{R}$$
$$g(x) = \frac{1}{x}, \; x \in \mathbb{R}, \; x \neq 0$$

(a) Sketch the graph of $y = h(x)$. *[2]*

(b) Find an expression for the composite function $h \circ g(x)$ and state its domain. *[2]*

Given that $f(x) = h(x) + h \circ g(x)$,

(c) (i) find $f'(x)$ in simplified form;

(ii) show that $f(x) = \frac{\pi}{2}$ for $x > 0$. *[7]*

Nigel states that f is an odd function and Tom argues that f is an even function.

(d) (i) State who is correct and justify your answer.

(ii) Hence find the value of $f(x)$ for $x < 0$. *[3]*

Please **do not** write on this page.

Answers written on this page
will not be marked.